Math 116 Section 04

Quiz 2 | Name ____________________
July 28, 2005
Instructor: Charles Cuell | Student Number ____________

All solutions are to be presented on the paper in the space provided. The quiz is open book. You can discuss the problem with others and ask the TA questions.

Expand the following sums. Example: $\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n$

(1)

$$\begin{aligned}\sum_{i=0}^{n-1} \frac{(-1)^i}{i+1} &= \frac{(-1)^0}{0+1} + \frac{(-1)^1}{1+1} + \frac{(-1)^2}{2+1} + \ldots + \frac{(-1)^{n-1}}{(n-1)+1} \\ &= 1 - \frac{1}{2} + \frac{1}{3} + \ldots + \frac{(-1)^{n-1}}{n}\end{aligned}$$

(2)

$$\sum_{k=2}^{n} \frac{e^{-k}}{n+k} = \frac{e^{-2}}{n+2} + \frac{e^{-3}}{n+3} + \frac{e^{-4}}{n+4} + \ldots + \frac{e^{-n}}{2n}$$

Rewrite the following using sigma notation:

(1) $\frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \ldots \frac{1}{99} = \sum_{i=7}^{99} \frac{1}{i}$

(2) $2^2 - 3^2 + 4^2 + \ldots - 99^2 = \sum_{i=2}^{99} (-1)^i i^2$

(3) $\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \ldots + \frac{n}{2^n} = \sum_{i=1}^{n} \frac{i}{2^i}$

$$\begin{aligned}\sum_{i=m}^{n} (2i - i) &= \sum_{i=m}^{n} i \\ &= \sum_{i=1}^{n} i - \sum_{i=1}^{m-1} \\ &= \frac{n(n+1)}{2} - \frac{(m-1)(m-1+1)}{2}\end{aligned}$$

$$= \quad \frac{n(n+1)}{2} - \frac{(m-1)m}{2}$$

Using the formula for the area under a curve, $A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$, calculate the area bounded by the lines $y = x + 1$, $y = 0$, $x = 0$ and $x = 2$.

$\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$.

Using right endpoints, $x_i^* = a + i\Delta x = 0 + i\frac{2}{n} = \frac{2i}{n}$ So,

$$\begin{aligned}
A &= \lim_{n\to\infty} \sum_{i=1}^{n} (1 + x_i^*)\,\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{2i}{n}\right) \frac{2}{n} \\
&= \lim_{n\to\infty} \frac{2}{n} \left(\sum_{i=1}^{n} 1 + \frac{2}{n} \sum_{i=1}^{n} i \right) \\
&= \lim_{n\to\infty} \frac{2}{n} \left(n + \frac{2}{n} \frac{n(n+1)}{2} \right) \\
&= \lim_{n\to\infty} \frac{2}{n} (n + n + 1) \\
&= \lim_{n\to\infty} \frac{2}{n} (2n + 1) \\
&= \lim_{n\to\infty} \left(4 + \frac{2}{n} \right) \\
&= 4
\end{aligned}$$